



(1-4) Absolute Value or Magnitude :

The absolute value of number is numerical size or magnitude. $|3| = 3$, $|0| = 0$, $|-5| = 5$

The absolute value or magnitude of a number x , denoted by $|x|$ (read the absolute value of x),

is defined by the formula: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

The connection between absolute values and distance gives us a new way to write formulas for intervals. The inequality $|a| < 5$ says that the distance from a to the origin is less than 5. This is the same as between 5 and -5 on the number line. In symbols:

$$|a| < 5 \leftrightarrow -5 < a < 5$$



The set of numbers a with $|a| < 5$ is the open interval from -5 to 5.

The general rule is this:

Relation between Intervals and Absolute Values

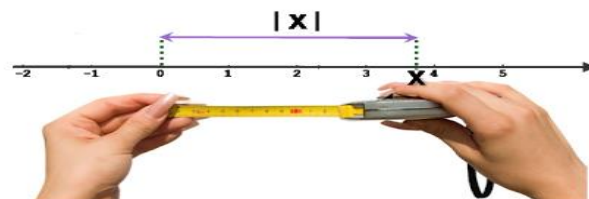
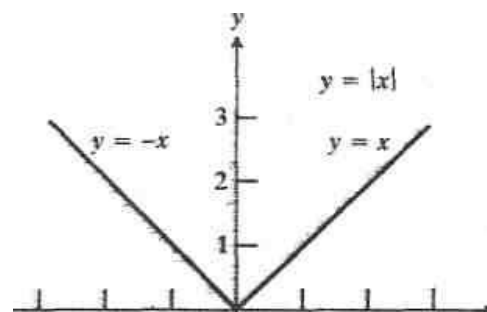
If D is any positive number, then:

$$|a| < D \leftrightarrow -D < a < D \quad (1)$$

$$|a| \leq D \leftrightarrow -D \leq a \leq D \quad (2)$$

The function $y = |x|$ is called *the absolute value function*.

Its graph lies along the line $y = x$ when $x \geq 0$, and along the line $y = -x$ when $x < 0$.





EXAMPLE 1: What values of x satisfy the inequality $|x - 5| < 9$?

Solution:

Change $|x - 5| < 9$ to $-9 < x-5 < 9$ {Eq. (1) with $a=x-5$ and $D = 9$ }

to $-9+5 < x < 9+5$ {Adding a positive number to both sides of an inequality gives an equivalent inequality. Adding 5 here isolated the z .}

or: $-4 < x < 14$.

The steps we just took are reversible, so the values of x that satisfy the inequality $|x - 5| < 9$ are the numbers in the interval $-4 < x < 14$.

EXAMPLE 2:

Solve the equation $|2x - 3| = 7$

Solution : The equation says that $2x - 3 = \pm 7$, so there are two possibilities:

a) $2x - 3 = 7 \rightarrow 2x = 10 \rightarrow x = 5$

b) $2x - 3 = -7 \rightarrow 2x = -4 \rightarrow x = -2$

The equation $|2x - 3| = 7$ has two solutions: $x = 5$ and $x = -2$

EX. 3: Solve the equation $|\frac{x}{5} - 2| = 1$

Solution : The equation says that $\frac{x}{5} - 2 = \pm 1$, so there are two possibilities:

a) $\frac{x}{5} - 2 = 1 \rightarrow \frac{x}{5} = 3 \rightarrow x = 15$

b) $\frac{x}{5} - 2 = -1 \rightarrow \frac{x}{5} = 1 \rightarrow x = 5$

The equation $|2x - 3| = 7$ has two solutions: $x = 15$ and $x = 5$

Exercise: solutions

1) $|x - 2| < 1/2$

1) $(3/2, 5/2)$

2) $|\frac{9x}{7} - 1| = 1$

2) $x = 14/9$ and $x = 0$



Slope (M)

M is the measure of the steepness of the line.

M in **mathematics** is the rate of change of **y** with respect to **x**.

How do we find Slope?

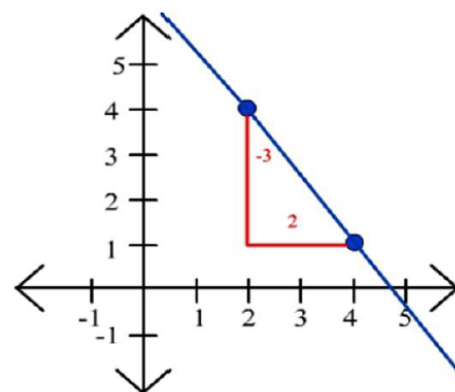
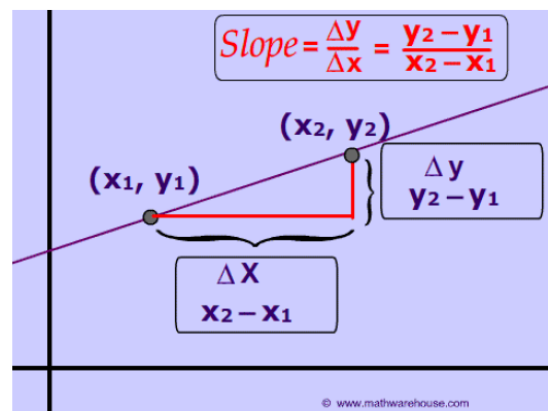
$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

The rise is -3 and the run is 2.

$$\text{Slope} = \frac{-3}{2}$$

The slope for the line above is $-\frac{3}{2}$.

It is a negative slope!



Point–Slope Equations

To write an equation for a line L that is not vertical, it is enough to know its slope m and the coordinates of a point $P_1(x_1, y_1)$ on it. If $P(x, y)$ is any other point on L (Fig. 1.24), then $x \neq x_1$ and we can write the slope of L as

$$\frac{y - y_1}{x - x_1} \quad (8)$$

We can then set this expression equal to m to get

$$\frac{y - y_1}{x - x_1} = m. \quad (9)$$

Multiplying both sides of Eq. (9) by $x - x_1$ gives us the more useful equation

$$y - y_1 = m(x - x_1). \quad (10)$$



EXAMPLE 5 Write an equation for the line that passes through point (2, 3) with slope $-3/2$.

Solution

$$\begin{aligned}
 y - y_1 &= m(x - x_1) && \left(\text{Start with the general point-slope equation, Eq. (11)} \right) \\
 y - 3 &= -\frac{3}{2}(x - 2) && \left(\text{Take } m = -3/2 \text{ and } (x_1, y_1) = (2, 3). \right) \\
 y &= -\frac{3}{2}x + 3 + 3 && \left(\text{Solve for } y. \right) \\
 y &= -\frac{3}{2}x + 6.
 \end{aligned}$$

EXAMPLE 6 Write an equation for the line through $(-2, -1)$ and $(3, 4)$.

Solution We first calculate the slope and then use Eq. (11).

$$m = \frac{-1 - 4}{-2 - 3} = \frac{-5}{-5} = 1.$$

The (x_1, y_1) in Eq. (11) can be either $(-2, -1)$ or $(3, 4)$:

With $(x_1, y_1) = (-2, -1)$	With $(x_1, y_1) = (3, 4)$
$y - (-1) = 1 \cdot (x - (-2))$	$y - 4 = 1 \cdot (x - 3)$
$y + 1 = x + 2$	$y - 4 = x - 3$
$y = x + 1.$	$y = x + 1.$
$\swarrow$ Same result $\swarrow$	

Either way, $y = x + 1$ is an equation for the line (Fig. 1.26).